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AUTHOR'S CLOSURE ON THE PAPER: "IL'IUSHIN'S POSTULATE AND RESULTING THERMODYNAMIC CONDITIONS ON ELASTO-PLASTIC COUPLING"

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If Professor Paglietti's discussion on [1] referred only to the old dispute on the cartesian decomposition of the total strain tensor into elastic and plastic components for finite deformations, it would be sufficient to mention Ref. [2] by Green and Naghdi which has closed the subject long ago. It seems, however, that there is a misunderstanding of the reference which the author of the discussion employs, thus the need for further elaboration here.

Professor Paglietti's basic argument is that $\dot{E}_p \neq 0$ for unloading $L \leq 0$. This is erroneous in view of the constitutive relation (11) of [1], thus the observations and results, following this argument in the discussion, are all unfounded. To begin with, the statement in the discussion that E_p is defined in [1] by

$$E = E_e + E_p \tag{1}$$

is wrong. A more careful examination of [1] would show that E_p is defined by its rate constitutive relation (11) and is eqn (14) in [1] which introduces an elastic strain tensor $E_e = E - E_p$. No particular kinematical interpretation has been adopted for E_p and E_e.

The work by Lee and Liu[3] (and a following paper by Lee[4]) is employed by Dr. Paglietti to advance his argument that $\dot{E}_p \neq 0$ for $L \leq 0$ if eqn (1) holds. It is believed that the statements in [3] are misunderstood and it will be shown that by no means the above argument can be derived from [3]. Following [4], the deformation gradient F is written as

$$F = F_e F_p \tag{2}$$

with respect to three configurations, the initial, the unstressed (not a continous one) and the current. Then, an elastic strain tensor $\bar{E}_e$ can be defined from

$$\bar{E}_e = \frac{1}{2}(F_e^T F_e - I) \tag{3}$$

with respect to the intermediate unstressed configuration.

Within the framework of the special kinematical interpretation of the three configurations, Green and Naghdi[2] show that they can identify a tensor E_p defined by

$$E_p = \frac{1}{2}(F_p^T F_p - I) \tag{4}$$

with the plastic strain tensor introduced in their general theory[5], which in fact is the plastic strain tensor introduced in [1]. Then, they can interpret

$$E_e = E - E_p \tag{5}$$

as "elastic" strain (as pointed out by Dr. N. Fox) valid for any elastic–plastic continuum, measuring the change in the lengths of line elements from the unstressed to the current configuration with respect to a convected system of coordinates. It is very important for the present closure to emphasize the $\bar{E}_e$ introduced by (3) is different from E_e introduced by (5). The former is defined with respect to the intermediate unstressed configuration while the latter is defined with respect to the initial configuration (since both E and E_p refer to the initial configuration as follows from eqns (2) and (4)). It is worth mentioning that at a later work by Naghdi and Trapp[6] within the kinematical interpretation of the three configurations, elastic and plastic strains are defined as in eqns (3) and (4) [eqns (9), (32), (34), (37) in [6]). The relation between E_e and $\bar{E}_e$ can be readily found. Using eqns (2)–(5) we have

$$E_e = \frac{1}{2}(F^TF - F_p{}^TF_p) = \frac{1}{2}F_p{}^T(F_e{}^TF_e - I)F_p = F_p{}^T\bar{E}_eF_p. \tag{6}$$

We can now quote from Ref. [3], page 22 which is used by Dr. Paglietti to advance his argument:

> "The strain is formally considered as the sum of elastic and plastic components, each of which satisfy the invariance properties of the total strain. This provides a simple summation law for elastic and plastic strain components in the range of finite strain. However, since strains are referred to the initial configuration, the elastic law, for example, on unloading, must involve the plastic-strain components through the geometry of the unstressed configuration, quite apart from the influence of plastic flow on the elastic characteristics of the material. Because of the inclusion of plastic strain in the elastic law, Naghdi and Green's theory takes care of the difficulty. However, by utilizing the unstressed configuration as a reference for elastic strains, the purely geometrical influence is eliminated, ..."

It follows that the authors of the above quotation are concerned about the "purity" of the elastic strain tensor E_e, supporting the argument that $\bar{E}_e$ is a "purer" elastic strain tensor than E_e in the sense that it measures macroscopically deformations associated with purely elastic microscopic mechanisms of the material, without the geometrical influence of the existing plastic deformations. We do not dispute here this argument. In fact in view of eqn (6), is obvious that upon unloading where $\dot{F}_p = 0$, the change of the "purer" $\bar{E}_e$ is compatible with the corresponding change of E_e only through the F_p which is a measure of the geometry of the unstressed configuration. What we do dispute however, is that from the above quotation can ever be concluded, as Dr. Paglietti supports, that the plastic strain tensor E_p is influenced by the elastic strain tensor E_e so that we have $\dot{E}_p \neq 0$ for $L \leq 0$. It is rather the inverse which is true, namely that the change of E_e in unloading is related at each plastic state to the change of $\bar{E}_e$ by the unchanging F_p as shown in eqn (6). The unfounded inversion of the meaning of the above quotation seems to be the basis for the misunderstanding.

Whatever the explanation for the misunderstanding may be, it is a fact that $\dot{E}_p = 0$ when $L \leq 0$, since this is the rate constitutive relation for E_p, and this is true whether or not we adopt the cartesian decomposition of E and/or the kinematical interpretation of the 3 configurations. To emphasize that, we refer once more to Refs. [5] and [6] where in the former the cartesian decomposition is assumed and in the latter no such decomposition applies, nevertheless always $\dot{E}_p = 0$ when $L \leq 0$.

Finally another small point in the discussion must be corrected. It is stated that the central relation

$$\frac{\partial}{\partial q_N}(\hat{\psi}_M - \hat{\psi}_T)\, r_N \geq 0 \tag{7}$$

in [1], is derived from Il'iushin's postulate and the second principle of thermodynamics. This is erroneous, because (7) was derived in [1] only from Il'iushin's postulate.

In closing I would like to thank Prof. Paglietti for his interest and discussion, which gave me the opportunity to elaborate on some points of common interest.

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